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The wire mesh regenerator of the Stirling cycle machine

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Abstract—Appropriately transformed, the classic analytical description of the counterflow regenerator lends itself to straightforward algebraic solution. Flushing ratios of any magnitude are accommodated, and all regimes of solution are described from initial blow through to cyclic equilibrium. The uniform particle velocity of the original treatment may be substituted by simple harmonic motion without complication, affording insight into conditions in the Stirling engine. Temperature recovery ratio is shown to be an inadequate guide to overall thermal performance. A more revealing index is derived by taking pumping loss into consideration. Design charts are presented.

BACKGROUND

ONE OF engineering's most enduring analytical problems [1], until its definitive solution [2, 3], has been that of describing the start-up and cyclic performance of the counter-flow thermal regenerator.

The regenerator is a matrix of porous solid cyclically heated and cooled by alternating currents of hot and cold fluid (Fig. 1). Amongst several applications it functions as the crucial, central component of the Stirling cycle machine.

An outline of the regenerator problem, and of the attempts which have been made at solution, may be found in ref. 4, which offers a novel analytical statement based in Lagrange coordinates, and which acquires specimen solutions with the aid of the computer. It has since been found possible [3] to take the analytical process forward without complication to an explicit statement of local, instantaneous difference between gas temperature and matrix temperature and thus, effectively, to a one-line solution. The rôle of the computer is then merely to make the substitutions required for display of specimen temperature profiles, and to process the solution repeatedly to give recovery ratio as a function of the operating parameters.

Earlier attempts at solution were so involved, and eventual solution so straightforward, that it is appropriate to seek an explanation for the contrast. This appears to be as follows: the analytical pioneers—principally Hausen [5] and Nusselt [6]—appear to have adopted a mechanical approach to solution at the expense of such physical realities as the fact that local, instantaneous heat exchange rate is at all times and locations related to local, instantaneous difference, ΔT, between wall and gas temperatures, and that ΔT is thus the 'natural' dependent variable of the problem. Most other analysts followed suit. The result was algebra of such complexity that the only solutions which could be obtained were those which ignored the *flushing* phase—i.e. that part of the blow involved in removing the residue of fluid from a previous blow. By contrast, the transformation which now permits ready solution is based firmly in the physics of the heat exchange phenomenon.

The transformation applies to an analytical description extended to take account of conditions representative of the practical Stirling machine. This paper obtains solutions taking account of:

- simple harmonic particle motion;
- low flushing ratios;
- specific matrix geometry—weave of mesh, wire diameter, pitch, porosity, etc. of wire material; and
- heat transfer and flow friction effects characterized

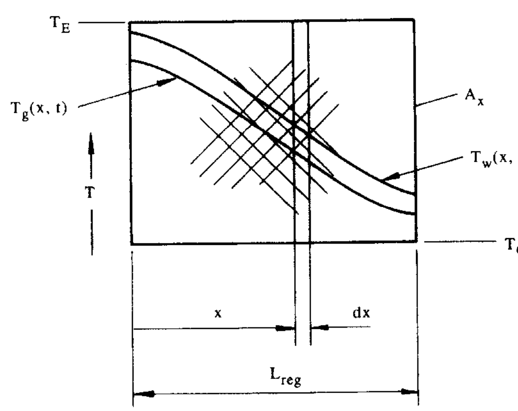

FIG. 1. Coordinate system for counterflow regenerator.

NOMENCLATURE

A, B constant terms, defined in text
A_x free-flow cross-sectional area [m^2]
a, b abbreviations for terms considered constant over an integration step, defined in text
c_p, c_v specific heat at constant pressure/volume [J kg^{-1} K^{-1}]
C_f friction factor = (equivalent wall shear stress)/$\frac{1}{2}\rho u^2$
c common specific heat (incompressible fluid) [J kg^{-1} K^{-1}]
d_w diameter of wire of regenerator gauze [m]
g mean mass velocity, ρu [kg m^{-2} s^{-1}]
H enthalpy per blow [J]
h coefficient of convective heat transfer [W m^{-2} K^{-1}]
l, L length [m]
L_{ref} a reference length—may be L_{reg} (below) [m]
L_{reg} overall physical length of regenerator packing [m]
m_w mesh number—number of wires/unit length [m^{-1}]
N_{MA} $\omega L_{ref}\sqrt{(RT_{ref})}$
N_{SG} Stirling number, $p_{ref}/\omega\mu$
N_{RE} characteristic Reynolds number = $N_{SG}N_{MA}^2$
N_{re} local, instantaneous Reynolds number = $4\rho u r_h/\mu$
N_{st} Stanton number = h/gc_p
N_T characteristic temperature ratio, T_E/T_C (note that this is the inverse of the traditional definition of temperature ratio for Stirling cycle machines, but consistent with more recent notation)
N_{TCR} thermal capacity ratio = $\rho_w c_w/\rho c$ for incompressible fluid, or $T_{ref}\rho_w c_w/p_{ref}$ for compressible fluid. Respective numerical values differ by a factor $(\gamma-1)/\gamma$
NTU 'number of transfer units', $N_{st}x/r_h$, $N_{st}L_{reg}/r_h$
n_r number of regenerator sub-divisions
P wetted perimeter [m]
p absolute pressure [Pa]
p_{ref} reference pressure [Pa]
Q' volume flow rate [m^3 s^{-1}]
R gas constant for specific gas [J kg^{-1} K^{-1}]
r_h hydraulic radius, free-flow area/wetted perimeter [m]
t time [s]
T_C, T_E constant, uniform temperatures of gas entering from right and left extremity of regenerator, respectively [K]
U 'utilization factor' employed by Hausen and defined in text, $U = \Pi/\Lambda_{Hausen}$
u velocity [m s^{-1}]
$\mathbf{u}$ normalized velocity, $u/\omega L_{ref}$
x length coordinate [m].

Greek symbols
γ specific heat ratio, c_p/c_v
ε regenerator 'inefficiency', defined in text
η temperature recovery ratio, defined in text
Λ reduced length or 'flushing ratio'—ratio (gas particle excursion amplitude)/L_{reg}
Λ_{Hausen} 'reduced length' employed by Hausen—defined in text—equivalent to NTU
λ normalized length/distance, l/L_{ref}, x/L_{ref}
λ_h normalized hydraulic radius, r_h/L_{ref}
Π reduced period employed by Hausen and defined in text
ρ density [kg m^{-3}]
ϕ crank angle (or dimensionless time) = ωt [rad]
ω angular frequency = $2\pi n_s$ [rad s^{-1}]
$¶_v$ volumetric porosity $\approx 1-1/4\{\pi m_w d_w\}$ for rectangular wire gauze.

Subscripts
g gas or fluid
i ith location along axis representing time (or dimensionless time)
j jth location along axis representing distance (or dimensionless distance)
Hausen as used by Hausen
w wall, wire
_ (underscore) mean value during finite interval of numerical integration step.

in terms of local, instantaneous Reynolds number, N_{re}.

Figure 1 represents the regenerator in its conventional siting between heater and cooler of the planar equivalent Stirling machine. An element dx long at location x from an origin at the expansion end is identified for the purposes of control-volume analysis.

There are several, independent sources of evidence, e.g. ref. [7], that the angular phase difference between a surface temperature disturbance and centre-line response in the individual regenerator wire of the typical, high-performance Stirling machine at rated operating conditions is negligible, being of the order of 0.2° of crankshaft rotation. On this basis, it is proposed to treat transient response of the system in isolation from conditions internal to the individual wire.

TRANSIENT THERMAL RESPONSE

Assumptions

Fluid entering from the left (expansion exchanger) does so at temperature T_E; that entering from the right (compression exchanger) at T_C.

Within the fluid there is zero diffusion and dispersion of temperature information parallel to flow direction, but instantaneous diffusion of temperature information perpendicular to that direction (see previous section). Hence, fluid properties are uniform in a plane perpendicular to flow direction, as is matrix temperature.

Fluid flow is simple harmonic. Extension of the treatment to flow patterns computed for specific Stirling machines is straightforward.

The matrix is stacked from rectangular-woven wire screens having wire diameter d_w and mesh number m_w, volumetric porosity, $¶_v \approx 1-1/4\{\pi m_w d_w\}$ [7] and corresponding hydraulic radius, $r_h/d_w = 1/4\{¶_v/(1-¶_v)\}$. Wire material properties may be functions of temperature in the fashion of $c_w = c_{w_{ref}}(1+a_c T/T_{ref})$.

Heat transfer coefficient, h, and friction factor, C_f, are local, instantaneous values defined in terms of steady-flow correlations for the wire mesh in question by $N_{st} = N_{st}(N_{re}N_{pr}^{2/3})$, $C_f = C_f(N_{re})$, N_{re} being derived from local, instantaneous ρ, u.

Defining equations

The treatment follows the algebra of ref. 3: with A_x for cross-sectional free-flow area, and P for wetted perimeter the energy equation for the gas element is:

$$h[T_w(x,t)-T_g(x,t)]P\,dx = \rho u c_p A_x[\partial T_g(x,t)/\partial x]\,dx + \rho c_v A_x\,dx[\partial T_g(x,t)/\partial t)]. \quad (1)$$

Equation (1) neglects kinetic energy effects, as is customary in treatments of the regenerator.

Denoting volumetric porosity of the matrix by $¶_v$, the energy balance for the solid material of the control volume may be written:

$$h\{T_g(x,t)-T_w(x,t)\}P\,dx = \rho_w c_w A_x\,dx \times \frac{(1-¶_v)}{¶_v}\frac{\partial T_w(x,t)}{\partial t}. \quad (2)$$

For the rectangular-woven wire screen $¶_v \approx 1-1/4\{\pi m_w d_w\}$ and equation (2) takes account appropriately of wire diameter, d_w, and mesh number, m_w.

The novelty of the present treatment resides largely in working in terms of local, instantaneous temperature *difference*, $\Delta T(x,t) = T_g(x,t)-T_w(x,t)$. Omitting the subscripts for brevity and subtracting T_w from each isolated occurrence of T_g (and adding again):

$$-h\Delta T P\,dx = \rho u c_p A_x(\partial\Delta T/\partial x+\partial T_w/\partial x)\,dx + \rho c_v A_x\,dx(\partial\Delta T/\partial t+\partial T_w/\partial t).$$

N_{st} is defined as $h/\rho|u|c_p$. This is a modulus, whereas flow direction takes alternating sign. Rearranging and taking account of the possibility of positive and negative u:

$$-\{N_{st}/[r_h\,\mathrm{sign}\,(u)]\}\Delta T = \partial\Delta T/\partial x+\partial T_w/\partial x + (1/\gamma u)(\partial\Delta T/\partial t+\partial T_w/\partial t). \quad (3)$$

Treating equation (2) in a similar way and inverting:

$$(1/\gamma u)\partial T_w/\partial t = \{N_{st}/[\gamma r_h\,\mathrm{sign}\,(u)]\}\Delta T\frac{¶_v}{(1-¶_v)}\frac{\rho c_p}{\rho_w c_w}. \quad (4)$$

Equation (4) is substituted into equation (3):

$$-\{N_{st}/[r_h\,\mathrm{sign}\,(u)]\}\left\{1+\frac{¶_v}{[\gamma(1-¶_v)]}\frac{\rho c_p}{\rho_w c_w} + \partial T_w/\partial x\right\}\gamma u\Delta T = \partial\Delta T/\partial t+\gamma u\,\partial\Delta T/\partial x. \quad (5)$$

From the definition of the substantial derivative, the right-hand terms, in combination, denote the total change in ΔT in the direction γu in the time-distance plane. All of the milestone analyses [2, 5, 6, 8] infer an *incompressible* fluid: in the work of Hausen [5] and Nusselt [6] this shows up in the constancy of u and ρ; in the Heggs and Carpenter approach [2], and in that of Miyable *et al.* [8] the condition is expressed in use of c_p in both convective ($\partial/\partial x$) and $\partial/\partial t$ terms (γ replaced by unity). Although the present treatment may be pursued without the need for this simplification, it is adopted with a view to showing that the classic formulation lends itself to ready algebraic solution. With γ accordingly replaced by unity, equations (5) becomes:

$$-\{N_{st}/[r_h\,\mathrm{sign}\,(u)]\}\left[1+\frac{¶_v}{(1-¶_v)}\frac{\rho c_p}{\rho_w c_w} + \partial T_w/\partial x\right]u\Delta T = \partial\Delta T/\partial t+u\partial\Delta T/\partial x.$$

The first term on the right is now self-evidently the substantial derivative, D/dt, inviting re-writing as

$$\{N_{st}/[r_h\,\mathrm{sign}\,(u)]\}u\left[1+\frac{¶_v}{(1-¶_v)}\frac{\rho c_p}{\rho_w c_w}\right]\Delta T + \partial T_w/\partial t = -D\Delta T/dt. \quad (6)$$

Equation (6) places no restriction on particle displacement relative to regenerator length, and to this extent is apt for conditions in the Stirling machine where values close to unity are the norm.

At cyclic steady state in a Stirling machine of viable design (i.e. with an efficient regenerator) the swing in absolute T_w with time (crank angle) is finite, but the gradient, $\partial T_w(x,t)/\partial x$, is substantially constant and uniform for all x, t [7]. This is increasingly so with increasing thermal capacity ratio, $N_{TCR} = \rho_w c_w/\rho c_p$, and points to a special case of the classic regenerator problem (u uniform between switching) having a particularly simple solution. With $\partial T_w/\partial x$ truly constant, equation (6) is integrable analytically over a complete blow; where the variation in $\partial T_w/\partial x$ is significant, solution proceeds by integrating over small intervals using appropriate values of (variable) $\partial T_w/\partial x$ at each step. If $\partial T_w/\partial x$ is not strictly uniform and constant but nevertheless nearly so, the one-step integration process may be expected to yield an approximation for $\Delta T(x,t)$.

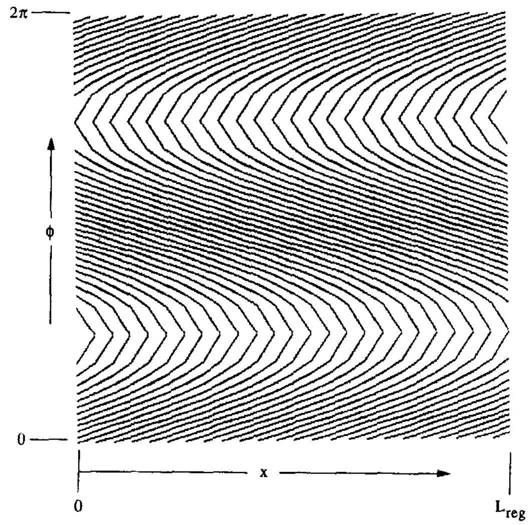

FIG. 2. Trajectories of selected fluid particles for flushing ratio, Λ, = unity.

Normalization—the concept of flushing ratio

The system of dimensionless parameters introduced by Hausen has become the norm. He defined [1] a reduced length, Λ_{Hausen}, essentially as:

$$\Lambda_{Hausen} = \frac{hA_w L_{reg}}{\rho u c_p A_x} = N_{st} L_{reg}/r_h. \tag{7}$$

Hausen's reduced length has little to do with length, and everything to do with *NTU*. The combination of variables defined in equation (7) will accordingly be denoted by this latter symbol. Hausen also defines two other parameters, namely, reduced period, Π, and utilization ratio, U:

$$\Pi = \frac{hA_x t_{blow}}{m_w c_w}, \tag{8}$$

$$U = \Pi/\Lambda = \frac{\rho u c_p A_x t_{blow}}{m_w c_w}. \tag{9}$$

U specifies the ratio of thermal capacity of the gas per pass to that of the matrix. The definition of U in terms of Π and Λ means that any two of the parameters suffice for the Hausen approach. Here we choose to continue in terms of *NTU* and U—the latter modified by inverting and denoting N_{TCR}:

$$N_{TCR} = \frac{\rho_w c_w}{\rho c}$$

when dealing with incompressible fluids, (10a)

$$N_{TCR} = \frac{T_{ref}\rho_w c_w}{p_{ref} c_p} \quad \text{for the compressible fluid.} \tag{10b}$$

Where the full form of the energy equation is retained, as here, *NTU* and N_{TCR} are insufficient to define the solution. The need for an additional parameter is readily appreciated from specimen particle trajectory diagrams. Figure 3 complements Fig. 2 with instances where: (a) particle amplitude is less than overall length of regenerator, L_{reg}; and (b) amplitude exceeds L_{reg}. In the former instance, a slug of fluid (shown hatched) oscillates within the matrix without exiting either end. In the latter, the matrix is completely flushed once per cycle, although not every particle which enters passes right through.

Figures 2 and 3 confirm that there are, in the general case, three solution domains; one for fluid which enters at temperature T_E at the left-hand extremity, and which exits left; a second for fluid which enters from the right at T_C and which exits right; and a third for any fluid which never exits either end. Solutions for the first two cases are determined by conditions at the boundaries, that of the third by initial values.

The parameter which determines how many solutions are in question is *flushing ratio*. Hausen's symbol Λ is taken over to denote this quantity, defined as the ratio [particle amplitude]/L_{reg}.

As regards normalized forms of dependent and independent variables, earlier treatments [4, 7] have established a system including crank angle (or dimensionless time) $\phi = \omega t$, and dimensionless velocity normalized by reference to angular frequency, n_s, on the basis that this latter variable is of more immediate engineering significance than angular frequency, ω. However, the expedient results in appearance of constant multipliers 2π. In this paper, ω will be used in the construction of normalized variables previously involving n_s. Thus, speed parameter (or characteristic Mach number, N_{MA}) becomes $\omega L_{ref}/\sqrt{(RT_{ref})}$, Stirling number, N_{SG}, becomes $p_{ref}/(\omega\mu)$ and dimensionless velocity, $\mathbf{u}$, $= u/\omega L_{ref}$. Conversion from the 'engineering' quantity *rpm* to ω is readily achieved on the basis that $\omega \approx rpm/10$.

Making equation (7) dimensionless puts it straightway into a form ready for integration:

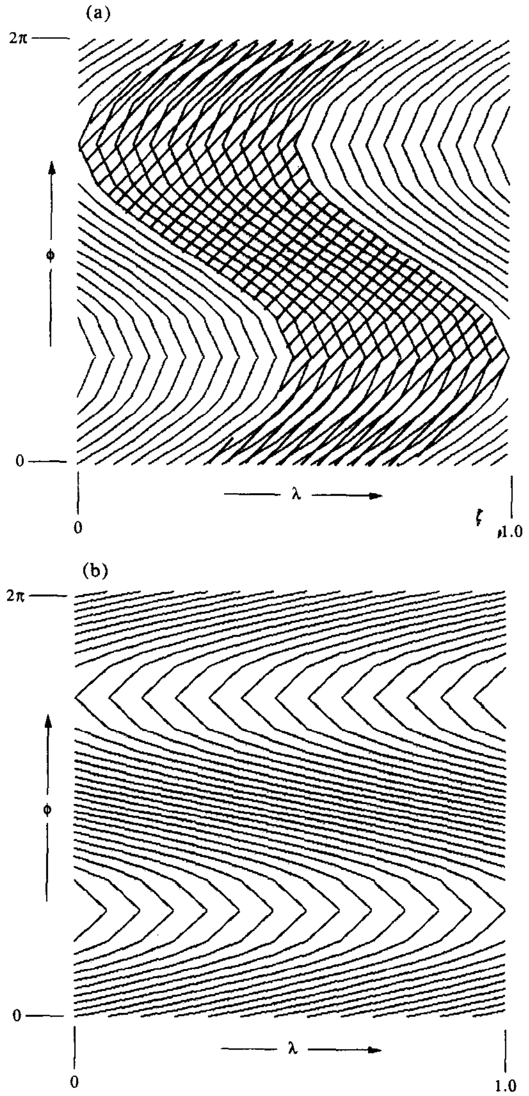

FIG. 3. Flushing ratios (a) less than and (b) greater than unity. In the former case, there are 3 distinct solution regimes, one each for fluid particles which enter at one end and leave by the same end, and a third for the slug of fluid which never exits either end. (a) Flushing ratio, Λ, $=0.5$, showing (hatched) slug of gas which oscillates within the matrix without leaving either end. (b) Flushing ratio, Λ, $=1.5$.

$$\int \frac{D\Delta t}{\Delta\tau + b/a} = -a \int d\phi$$

$$a = [NTU/\text{sign}(\mathbf{u})]\left[1 + \frac{\P_v}{(1-\P_v)N_{TCR}}\right]\mathbf{u}$$

$$b = \mathbf{u}\partial\tau_w/\partial\lambda.$$

Upon integration:

$$\frac{\Delta\tau + b/a}{\Delta\tau_0 + b/a} = \exp[-a(\phi - \phi_0)]. \quad (11)$$

At first sight, the parameter Λ is absent from the solution. In fact, it is present via the definition of $\mathbf{u}$: particle displacement, x, is given by $x = X\sin(\omega t)$, so that, as a fraction of L_{reg}, and in terms of dimensionless time, ϕ, $x/L_{reg} = \lambda = \Lambda\sin\phi$. $\mathbf{u} = d\lambda/d\phi = \Lambda\cos\phi$, and Λ is, indeed, a parameter of the solution.

Equation (11) may be contrasted in terms of its simplicity with the expression spanning many lines reached by Miyabe *et al.* on Laplace transformation of the energy equation. Even starker contrasts are with the daunting intricacy of solutions by Hausen and Nusselt—solutions which in any case embody compromise, which makes it virtually impossible to represent short blow times.

Consistent with the definition of the substantial derivative, equation (11) applies along particle trajectories (Heggs' and Carpenter's 'characteristics'). The 'characteristic' directions along which changes in $\Delta\tau$ are calculated are the particle paths inclined at $\pm 1/\mathbf{u}$ in the time–distance (ϕ–λ) plane.

It is not essential to proceed to the solution stage in order to be in a position to verify the self-evident physical meaning of equations (11):

- For thermal capacity ratio, $N_{TCR} = \rho_w c_w/\rho c_p$, effectively infinite (regenerator thermal capacity/unit volume greatly in excess of that of working fluid) $\partial T_w/\partial x$ is uniform and invariant by definition. The magnitude of a then reduces to $NTU/\text{sign}(\mathbf{u})$ and the solution for $\Delta\tau$ (local, instantaneous, dimensionless temperature *difference*) is that for the balanced, counterflow recuperator. *Absolute* gas temperatures are achieved by simply adding appropriate values of (invariant) wall temperature distribution, as intuition dictates.
- Under the above conditions, the solution for the gas is independent of that of the wall. It applies over any desired time step, $\phi - \phi_0$, provided account is taken of any intervening change in sign of $\mathbf{u}$.
- Under these same conditions ($N_{TCR} = \rho_w c_w/\rho c_p$, effectively infinite) $\Delta\tau$ at given location and time (or crank angle, ϕ) depends only on initial temperature difference (e.g. at entry), length of time ($\phi - \phi_0$) for which the particle has been in contact with the wall, and *NTU*.

Wall temperature

In the general case (thermal capacity ratio finite) wall temperature is a function of time and location. Normalizing equation (4) and taking into account the definition of thermal capacity ratio, N_{TCR}:

$$\partial\tau_w/\partial\phi = [NTU\,\text{sign}(\mathbf{u})]\Delta\tau\frac{\P_v}{(1-\P_v)N_{TCR}}\mathbf{u}. \quad (4a)$$

Equation (4a) confirms that, for sufficiently large N_{TCR}, $\partial\tau_w/\partial\phi$ = zero, signifying that wall temperature distribution, τ_w, does not vary with dimensionless time, ϕ.

For N_{TCR} and *NTU* finite, determination of τ_g and τ_w for all ϕ, λ requires simultaneous solution of equations (11) and (4a), calling for working in a combination of Lagrange and Euler coordinates. Appropriate choice

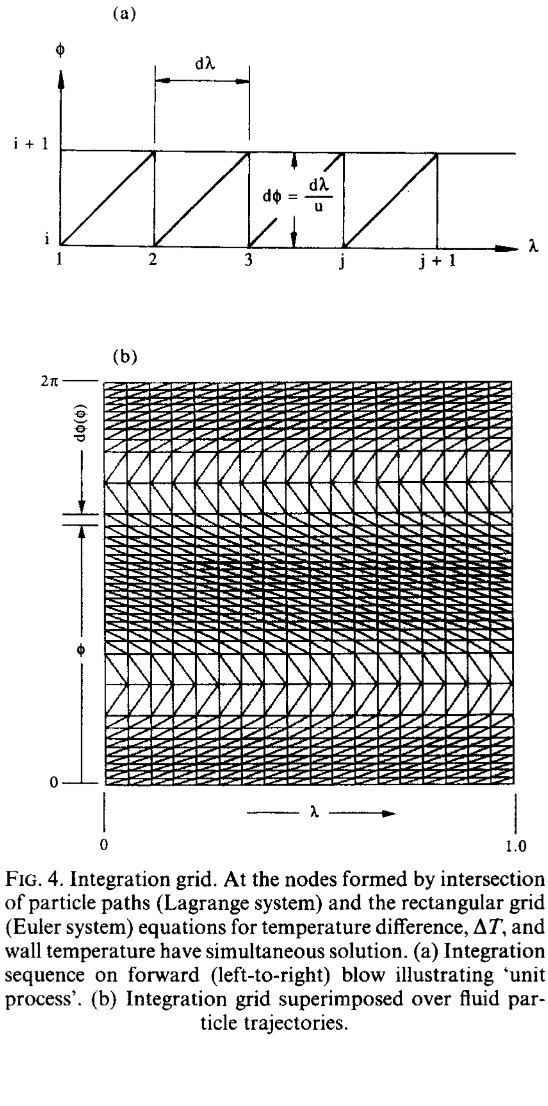

FIG. 4. Integration grid. At the nodes formed by intersection of particle paths (Lagrange system) and the rectangular grid (Euler system) equations for temperature difference, ΔT, and wall temperature have simultaneous solution. (a) Integration sequence on forward (left-to-right) blow illustrating 'unit process'. (b) Integration grid superimposed over fluid particle trajectories.

of integration grid renders straightforward an apparently daunting prospect.

UNIT PROCESS FOR INTEGRATION

Interior points

Equation (11) applies along a particle trajectory, while equation (4) for the wall applies at constant location, λ. With wall and gas temperatures known (or assumed) at a starting value of dimensionless time, ϕ, integration may proceed *without interpolation*: in Fig. 4a a particle at location i,j at time, $\phi(i,j)$ is at position $i+1,j+1$ after time increment $\Delta\phi$. With $\Delta\phi$ adjusted in relation to $\Delta\lambda$ such that the new location time $i+1$ coincides with the 'old' location of the adjacent particle at time i the 'new' wall temperature, $\tau_w(i+1,j+1)$ and 'new' value of temperature difference, $\Delta\tau(i+1,j+1)$, have simultaneous solution at point $i+1,j+1$. Figure 4b shows the integration grid set up to ensure coincidence of the points at which equations (11) and (4a) have common solution. The grid differs from the simple, Cartesian form only to the extent that increments in $\Delta\phi$ vary over a half cycle (i.e. between 0 and π rad).

Boundary conditions

The 'unit process' described above for integration applies at all points *except* the left-hand boundary on the forward (left–right) blow and at all points except the right-hand boundary on the reverse blow.

Defining the new temperature difference and the new wall temperature at the boundary is an essential part of the integration sweep, but is elementary. It is dealt with in the Appendix.

SPECIMEN TEMPERATURE PROFILES

With the Stirling cycle machine in mind, and with a view to speeding convergence of solution on cyclic steady state, wall and gas temperatures are set initially to a linear distribution from T_E at the expansion end to T_C at the compression end (in normalized form, from N_T at $\lambda = 0$ to unity at $\lambda =$ unity). Figure 5 illustrates the startup blow and first reverse for (a) gas and (b) wall for values of the parameters Λ, NTU, N_{TCR} stated with the figure. A thermal capacity ratio, N_{TCR}, of 10 is small for Stirling machine use, having been chosen to exaggerate matrix response. The profiles for the gas show clearly the discontinuity in temperature left at completion of the first blow which is pushed back into the matrix as a wave. Because the numerical phase of the solution is based on a finite number of particle trajectories, the discontinuities are of finite gradient: for an infinite number of trajectories they are infinitely steep.

Figure 6 shows temperature profiles after attainment of cyclic equilibrium (5 cycles for the parameters cited). The temperature envelopes follow the classic pattern, but internal to the envelopes may be seen the waves which arise at flow reversal and which survive the first stages of the following blow. The phenomenon is clearly on the temperature reliefs. For values of NTU and N_{TCR} typical of the Stirling machine, fluid temperature excursions are less pronounced, those of the wall even less so.

REGENERATOR INEFFECTIVENESS, ε

The standard measure of regenerator performance under given operating conditions is *recovery ratio*, η, defined as:

$$\eta = \frac{\int m' c_p (T_{out}^{actual} - T_{in})\, dt}{\int m' c_p (T_{out}^{actual} - T_{in})\, dt}. \quad (12)$$

The definition gives rise to two difficulties:

(a) The definition takes no account of pumping power, $\Delta p Q'$: for purposes of thermodynamic design it is necessary to take account of the trade-

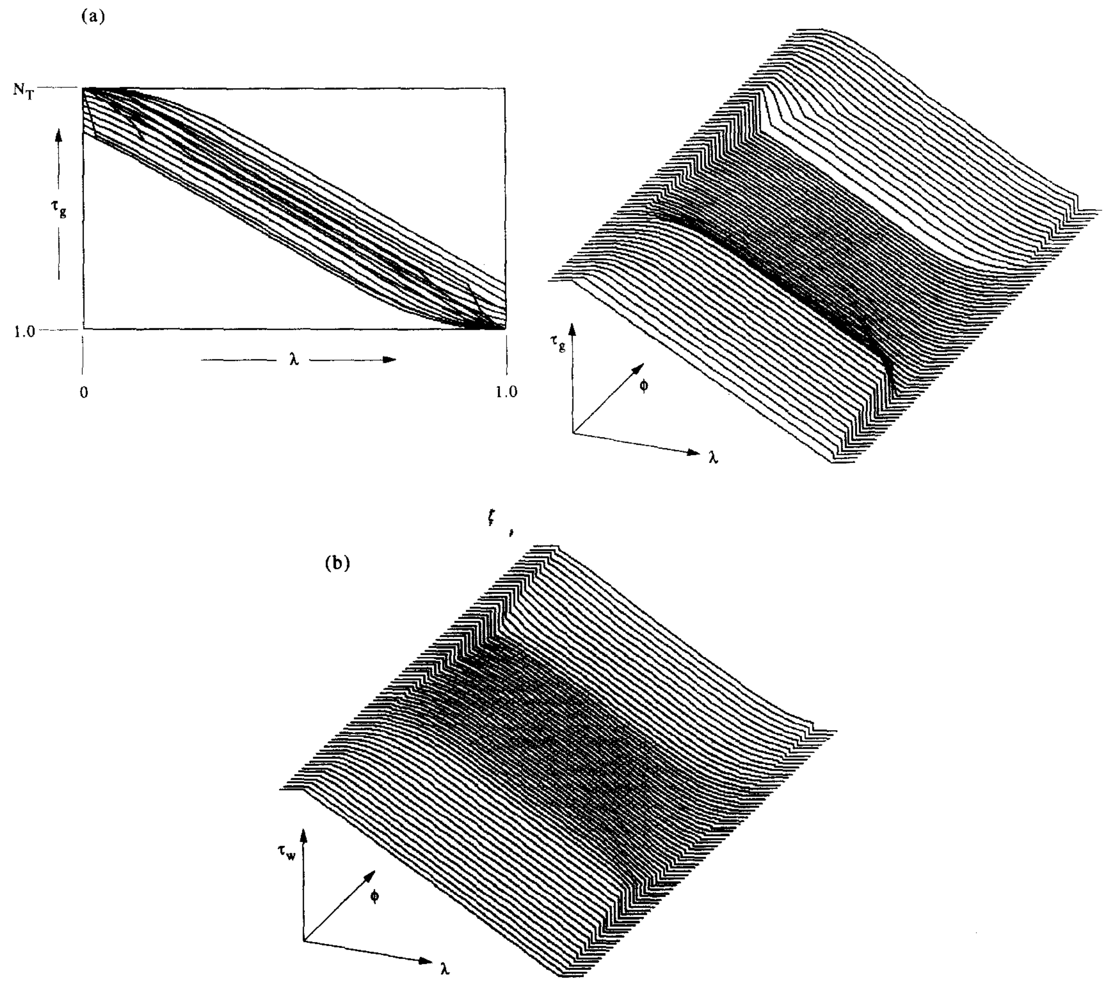

FIG. 5. Gas temperature reliefs for initial blow and first reverse. Flushing ratio, Λ, = unity, $NTU = 10$ and $N_{TCR} = 10$. (a) Temperature relief for fluid. (b) Temperature relief for wall.

off between increased recovery ratio and concomitant increased pumping power.

(b) Recovery ratios in the range 95–99% are of interest for Stirling cycle machines. Forms of display accommodating recovery ratios calculated for a wide range of *NTU*, etc. have poor resolution in this range.

With this in mind, *inefficiency* for the forward blow, ε, is defined in terms of two losses: (a) enthalpy *not* recovered and (b) pumping power. With C_f for friction factor:

$$\varepsilon = \frac{c_p \int \rho u A_x (T_{L,t} - T_C)\,dt + \int 1/2 \rho u^2 C_f (L_{reg}/r_h) A_x u \cdot dt}{c_p \int \rho u A_x (T_E - T_C)\,dt}. \quad (13)$$

The integral is conditional, applying only over those fractions of a complete revolution for which u is positive (towards the compression exchanger). An analogous expression applies to the reverse blow.

Now, for many heat exchanger surfaces [9] the heat transfer correlation $N_{st}N_{pr}^{2/3}$ vs N_{re} parallels that for friction factor, C_f vs N_{re}. For such cases it follows that curves of $N_{st}L/r_h$ (i.e. of *NTU*) are parallel to those of C_fL/r_h. This is not strictly true for the interrupted flow passages of the wire mesh regenerator, but is an approximation adequate for purposes of illustration. Setting $C_fL/r_h = A \cdot NTU$ in equation (13), therefore, and normalizing:

$$\varepsilon = \frac{\int \mathbf{u}(\tau_{1,\phi} - 1)\,d\phi + 1/2A \cdot NTU \cdot N_{MA}^2[(\gamma - 1)/\gamma] \int \mathbf{u}^3\,d\phi}{(N_T - 1)\int \mathbf{u}\,d\phi}. \quad (14)$$

γ appears through conversion of c_p to R in forming characteristic speed parameter, N_{MA}. It is left in place

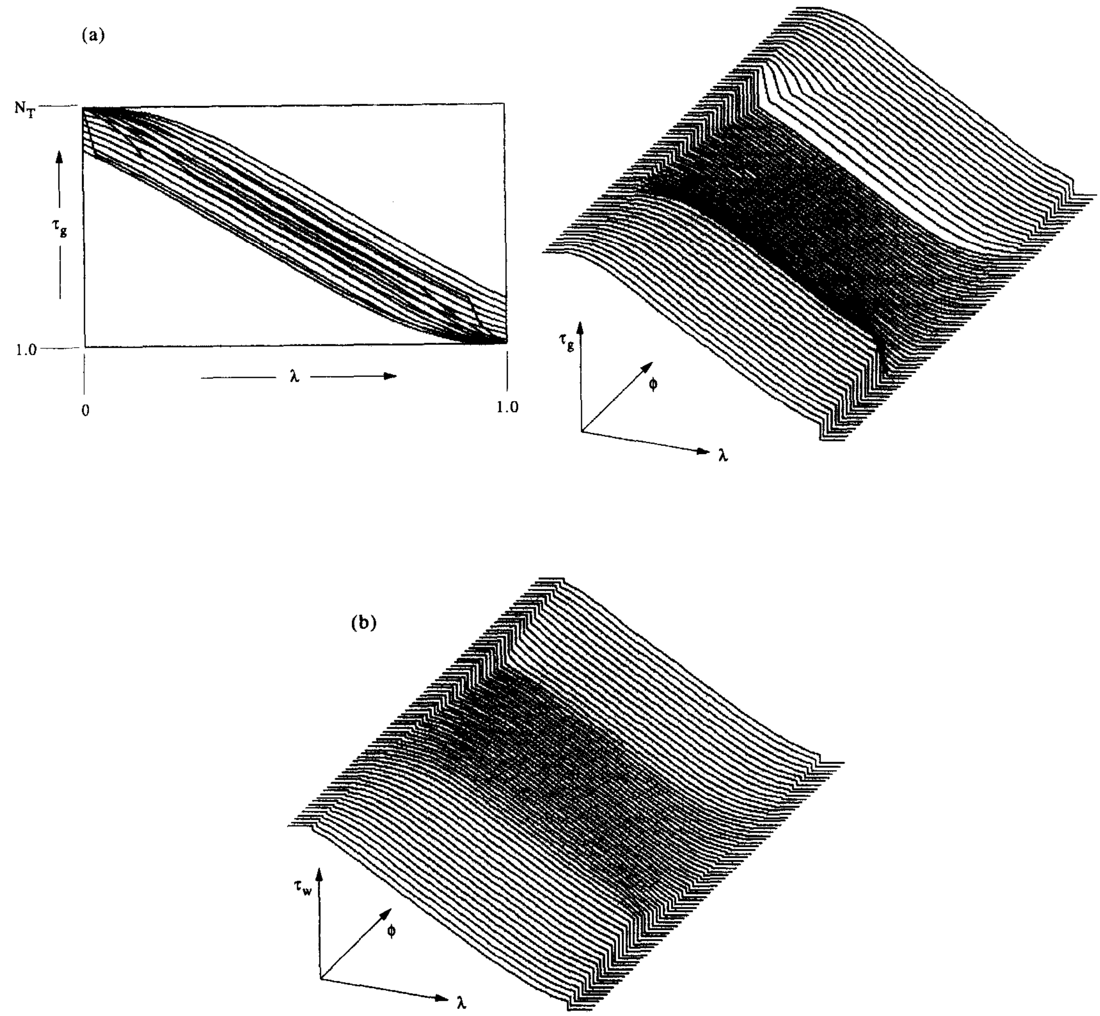

FIG. 6. Temperature profiles and reliefs for the conditions of Fig. 5 after attainment of cyclic steady-state. (a) Temperature profile and relief for fluid. (b) Temperature profile and relief for wall.

in equation (14) because, with sound speed infinite in the incompressible fluid, the second term in the numerator is otherwise zero.

u is a function of Λ, viz., $\mathbf{u} = \Lambda \cos \phi$, so that inefficiency is a function of the form:

$$\varepsilon = \varepsilon(N_T, \gamma, \P_v, \Lambda, NTU, N_{TCR}, N_{MA}). \quad (14a)$$

For a given Stirling machine, i.e. for given temperature ratio, N_T, given working fluid, γ, for given volumetric porosity, $\P_v$, and given speed parameter, N_{MA}, inefficiency, ε is a function of Λ, NTU and N_{TCR}.

To highlight the influence of pumping loss, N_{MA} is provisionally set equal to zero (no flow resistance). Figure 7a–c displays inefficiency, ε, for flushing ratios, Λ, of 0.5, 1.0 and 1.5, respectively. Independent variable is NTU and N_{TCR} is parameter. For $\Lambda \leqslant$ unity, losses measured in terms of temperature recovery are zero for NTU = zero, since the fluid exits with temperature unaltered since entry. Accordingly, both sets of curves for $\Lambda \leqslant$ unity show ε falling to zero for $NTU <$ unity. For $\Lambda >$ unity, some particles at least sweep the entire matrix, and inefficiency has finite values for NTU small and zero, as confirmed in Fig. 7c. All sets of curves show inefficiency rising initially with increasing NTU, but eventually falling, the fall-off following intuitive expectation.

Figure 8 corresponds to Fig. 7 except that N_{MA} has been set equal to 0.02—a value representative of operation at rated conditions for the Philips MP1002CA air engine. The analysis has been formulated in such a way that increase in NTU means corresponding increase in C_f. After initially following the trends of respective Fig. 7 at low NTU, friction effects begin to dominate for $NTU > 10$, to the extent that inefficiency increases linearly with increasing NTU.

The range of N_{TCR} covered by the curves is small—from 4.5 to 9.5 in the case of Fig. 7, and from 2.5 to 7.5 for Fig. 8. These uncharacteristically small values are chosen for the fact that the associated variation has a marked effect on inefficiency. As N_{TCR} increases from a value of 10 there is a gradual decrease in ε

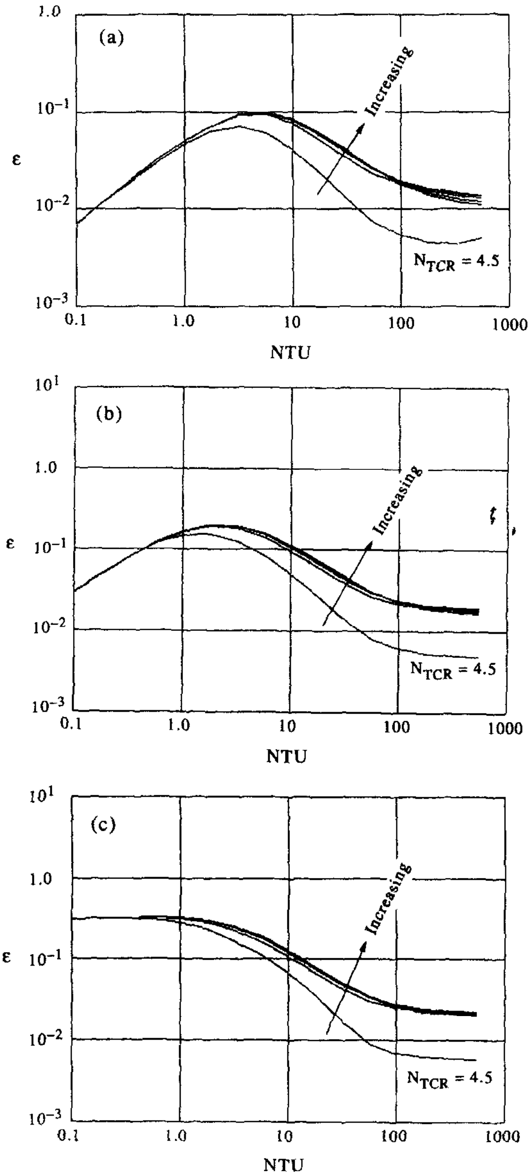

FIG. 7. Regenerator inefficiency, ε, plotted against NTU with thermal capacity ratio, N_{TCR}, as parameter. $N_{MA} = 0.0$ throughout, so pumping losses are artificially suppressed. (a) Flushing ratio, Λ, ≈0.5. (b) Flushing ratio, Λ, =unity. (c) Flushing ratio, Λ, = 1.5.

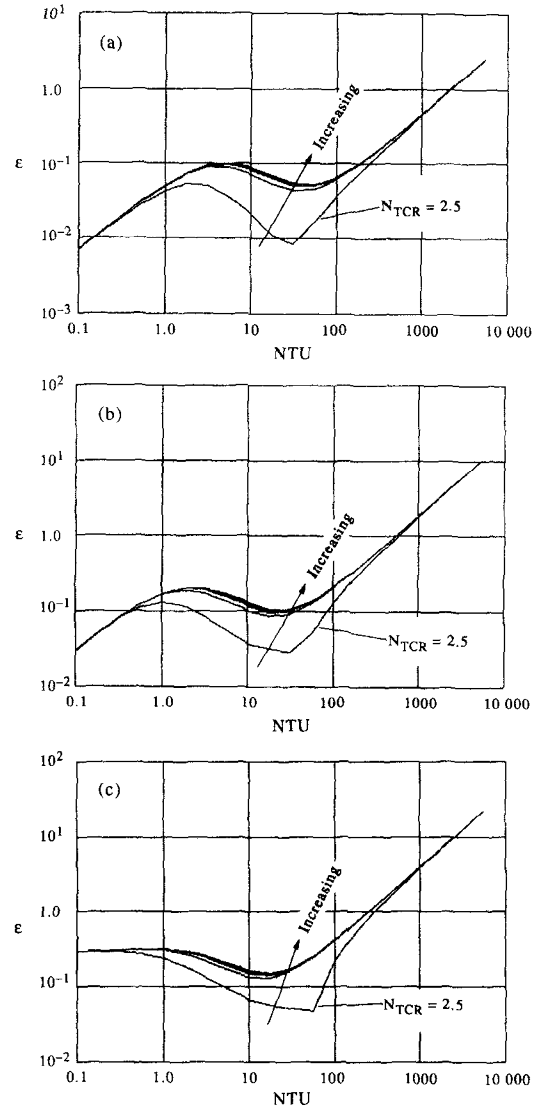

FIG. 8. Inefficiency, ε, plotted against NTU with thermal capacity ratio, N_{TCR}, as parameter. $N_{MA} = 0.02$ as for Philips MP1002CA air engine at rated operating point. (a) Flushing ratio, Λ, =0.5. (b) Flushing ratio, Λ, =unity. (c) Flushing ratio, Λ, = 1.4.

at given NTU. There is little advantage, according to the present treatment, in values of N_{TCR} in excess of 100.

The flushing ratio of Fig. 8c best represents practical values. Corresponding optimum NTU lies somewhere between 40 and 60, inviting comparison with findings of Miyabe *et al.* [8] : relying on indicated performance of a Stirling engine of their own design, those authors found optimum NTU to lie between the 65 and 128 afforded by gauzes of 100 and 150 mesh, respectively. The finding was consistent with predictions of their analytical approach, which, however, considered only the forward blow (i.e. which did not attain cyclic equilibrium), and which dealt with pumping loss separately from thermal recovery ratio.

CONCLUSIONS

* The classic regenerator problem in complete form, i.e. retaining the $\partial/\partial t$ term, is capable of general solution for simple harmonic particle motion for flushing ratio of any magnitude.
* An integration algorithm appropriate to solution of the temperature difference equation (Lagrange formulation) may be intercalated with an apt solution algorithm for the solid matrix heat balance (Euler) in a way which obviates interpolation between the two coordinate systems.
* A regenerator 'inefficiency', ε, may be defined

which takes into account pumping power as well as incomplete temperature recovery. The dependence of inefficiency, ε, on the principal operating parameters, number of transfer units, NTU, thermal capacity ratio, N_{TCR}, and flushing ratio, Λ, has been explored.

* Regenerator performance expressed in the traditional fashion, viz., terms of temperature recovery ratio alone is meaningless as an indication of suitability for use in the Stirling cycle machine.
* Charts are now available which permit selection of the operating conditions, NTU, N_{TCR} and Λ, which, according to the present theory, afford minimum inefficiency.
* The treatment is capable of ready extension to cover compressible flow, and to take account of the temperature dependence of matrix properties.

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APPENDIX

Boundary conditions (from Ref. 3)

Reference to Fig. 4 will confirm that, on the left–right blow, a sweep of unit processes leads to solutions at all 'new' points $j, i+1$ except the entry point $1, i+1$. This is the point at which gas enters left at constant, uniform temperature

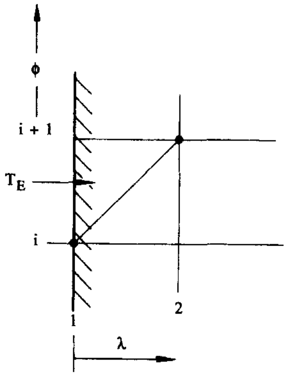

FIG. A1. Application of finite difference algorithm at left-hand boundary. Right-hand case is mirror image.

$T_g(0, t) = T_E$, or, in normalized form, $\tau(1, i+1) = N_T$. Likewise, on the reverse blow, all temperatures at $i+1$ are computed except that at $j = n_r$. Accordingly, algorithms are required to cope with these two points as exceptions to the unit process.

Equation (4a) is first abbreviated to:

$$\Delta\tau_w/\Delta\phi \approx B\Delta\tau \qquad (A1)$$

where, for the case in question, the terms now represented by the symbol B are, indeed, constant, and where, for more general cases, they may be treated as constant over time interval $\Delta\phi$. By the definition of the variable $\Delta\tau = \tau_g - \tau_w$ applied to the forward blow at $\lambda = 0$ (Fig. A1):

$$\Delta\tau(1, i+1) = N_T - \tau_w(1, i+1).$$

Integration of equation (A1) at $\lambda = 0$ calls for the mid-interval value of $\Delta\tau$, viz., $\underline{\Delta\tau}$:

$$\underline{\Delta\tau}(j, \underline{i}) = 1/2[N_T - \tau_w(1, i+1) + \Delta\tau(1, i)].$$

From equation (A1):

$$\tau_w(1, i+1) = \tau_w(1, i) + \Delta\phi B\,\underline{\Delta\tau}.$$

Substituting this most recent expression into that preceding and making $\tau_w(1, i+1)$ explicit:

$$\tau_w(1, i+1) = \frac{\tau_w(1, i) + 1/2B\Delta\phi[\Delta\tau(1, i) + N_T]}{1 + 1/2B\Delta\phi}. \qquad (A2)$$

The comparable expression $\tau_w(n_r, i+1)$, derived in similar fashion, is:

$$\tau_w(n_r, i+1) = \frac{\tau_w(n_r, i) + 1/2B\Delta\phi[\Delta\tau(n_r, i) + 1]}{1 + 1/2B\Delta\phi}. \qquad (A3)$$

Special cases (after Ref. 3)

Equation (11) is undefined for $NTU = 0$ because b/a becomes infinite. Provided integration step size is small (i.e. provided advantage is *not* going to be taken of N_{TCR} = large to integrate over an entire blow) this may be dealt with by expanding on the assumption that the numerical value of exponent $a(\phi - \phi_0)$ is small:

$$\Delta\tau + b/a = (\Delta\tau_0 + b/a)\exp[-a(\phi - \phi_0)]$$

$$\approx (\Delta\tau_0 + b/a)[1 - a(\phi - \phi_0)].$$

Expanding and cancelling the two b/a:

$$\Delta\tau \approx \Delta\tau_0 - (\partial\tau_w/\partial\lambda)\mathbf{u}(\phi - \phi_0). \qquad (A4)$$